Towards Tensor Regression with Applications in Neuroimaging Data Analysis

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Objectives & Motivations

Objectives:

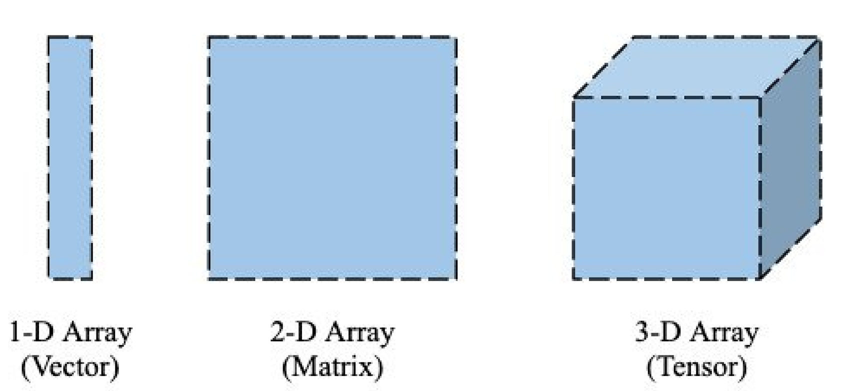
- Formulate a regression problem by treating neuroimaging data (e.g. EEG, fMRI) as covariates and clinical outcome as response
- Understand the neural development of normal brains and brains with disorders through neuroimaging data

Motivations:

- Classical generalized linear models (GLMs) use vectors as covariates
- Modern neuroimaging modalities generate covariates that are multidimensional (tensors)
- Vectorizing neuroimaging data can be million-dimensional, making computation infeasible
- Vectorizing destroys inherent spatial and temporal structure of tensor that hold valuable information

Preliminaries

Tensors:



Tensors are multidimensional arrays:

- Formally denoted as $\mathbf{X} \in \mathbb{R}^{D_1 \times D_2 \times \ldots \times D_N}$
- Generalizations of vectors and matrices

Generalized Linear Model:

$$y = \langle X, \beta \rangle + \epsilon,$$

where $y \in \mathbb{R}^{P \times 1}$, $X \in \mathbb{R}^{P \times N}$, $\beta \in \mathbb{R}^{N \times 1}$

Remarks:

- P denotes the sample size of X
- N denotes the dimensions of the vectorized form of X and β

Kronecker Product:

$$A = [a_1\ a_2\ \ldots\ a_M]^T \in \mathbb{R}^{M \times 1}$$
$$B = [b_1\ b_2\ \ldots\ b_N]^T \in \mathbb{R}^{N \times 1}$$
$$A \otimes B = [a_1 B\ a_2 B\ \ldots\ a_M B]^T \in \mathbb{R}^{MN \times 1}$$

Methodology

Problem Formulation:

- Let $\beta \in \mathbb{R}^{N \times N}$ denote the weight parameters in our regression model
- Let $\vec{\beta} = vec(\beta)$, where $vec(\cdot)$ is the function that vectorizes a multidimensional array
- If $\beta = ab^T$, where $a \in \mathbb{R}^{N \times 1}$ and $b \in \mathbb{R}^{N \times 1}$, then $\vec{\beta} = a \otimes b$, that is, β admits a Kronecker structure

By exploiting the Kronecker structure of our parameters, we rewrite our regression model formulation as the following:

$$y = \langle X, (\beta_1 \otimes \beta_2) \rangle + \epsilon$$

Solution:
To solve, formulate into an optimization problem:

$$\arg\min_{\beta_1, \beta_2} \frac{1}{N} \| Y - X(\beta_1 \otimes \beta_2) \|_F^2 \tag{1}$$

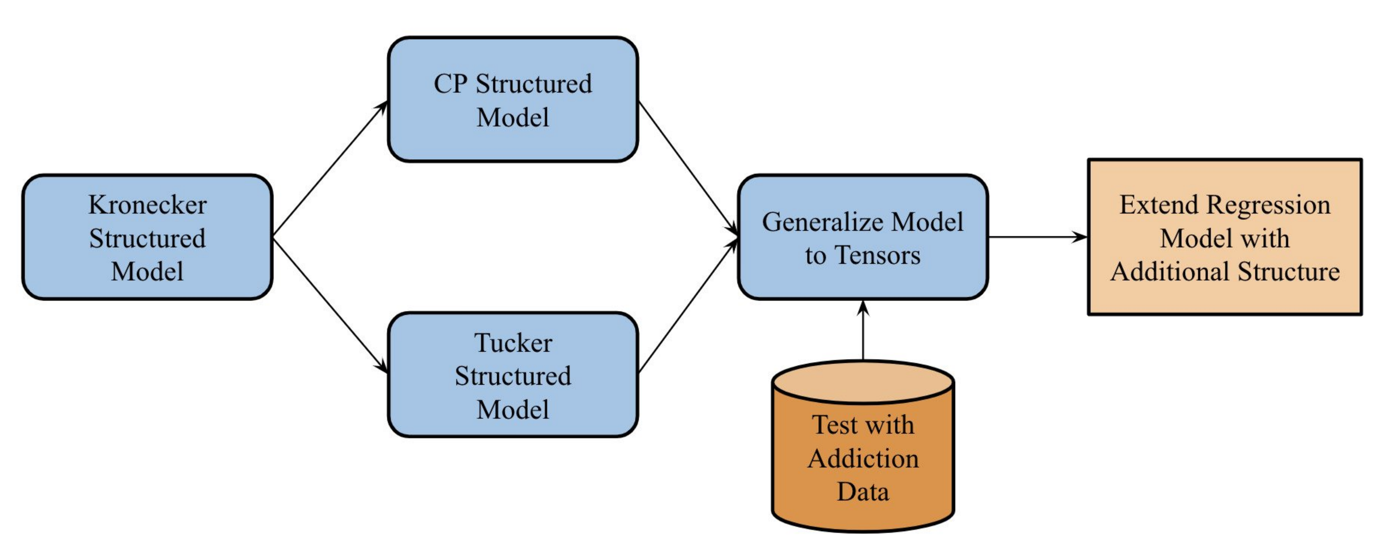
Remarks:

- Our model simplifies the GLM by solving for two parameters of dimensions $(N \times 1)$ than solving for one parameter of dimensions $(N^2 \times 1)$
- Can solve for β_1 and β_2 using an alternating minimization method or by gradient descent

Future Work & Plans

- Imposing a Kronecker structure to our regression parameters allows us to solve for less parameters more efficiently
- We can generalize this Kronecker model to tensor regression models with high-dimensional covariates

Roadmap:

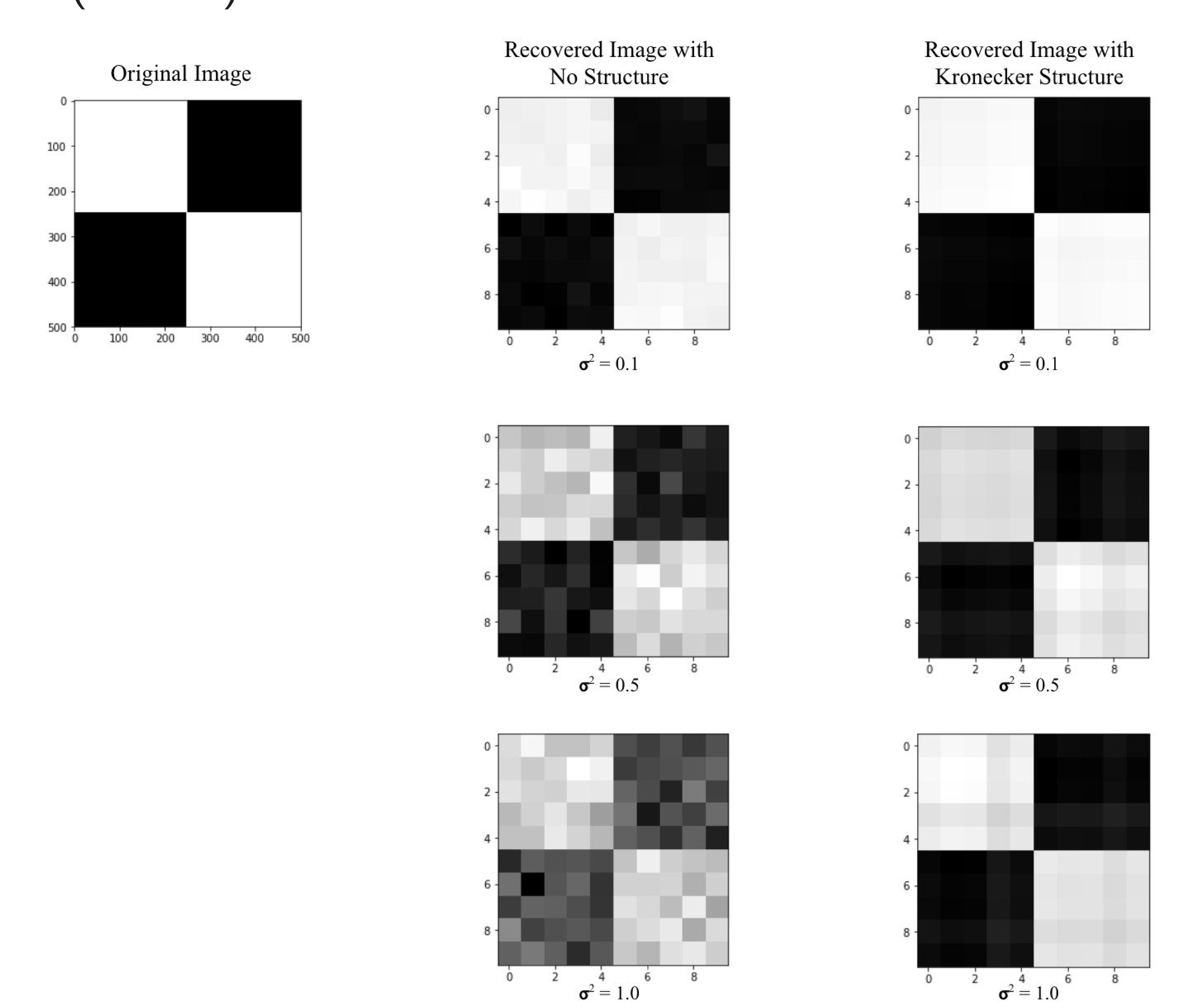


Results

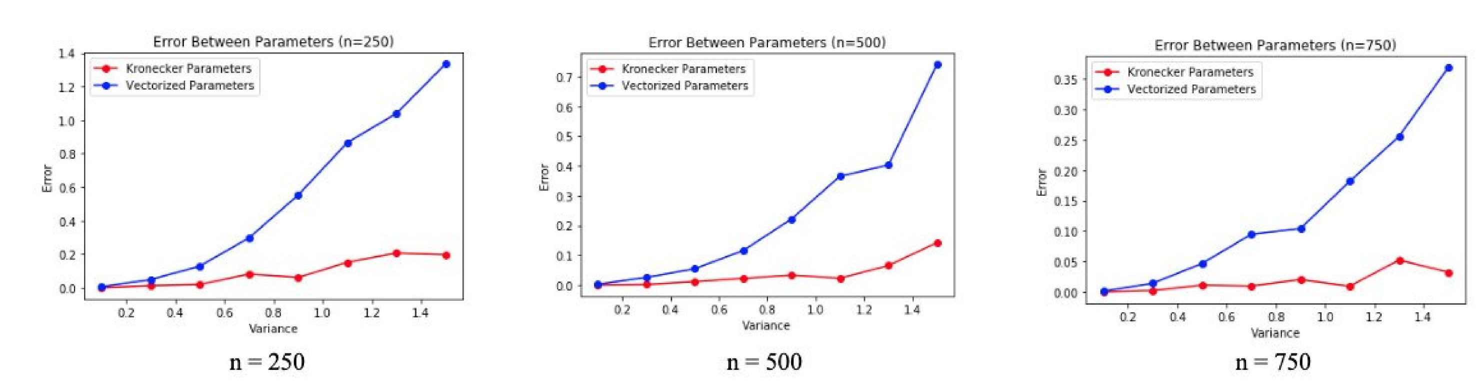
Goal: Observe the true signal region in β using synthetically generated data (x_i, y_i)

- Simulated n=250, 500, 750 (sample size) univariate responses y_i according to the Kronecker structured model
- $\beta \in \mathbb{R}^{10 \times 10}$ is a 2-dimensional image
- $X \in \mathbb{R}^{750 \times 100}$ is drawn i.i.d. from $N(0,1)$, where 100 is the vectorized dimensions of β
- ϵ is drawn from $N(0, \sigma^2)$ with different σ^2

Results (n=750):



Plot of Errors (n=250, 500, 750):



References

[1] Hua Zhou, Lexin Li, Hongtu Zhu. Tensor Regression with Applications in Neuroimaging Data Analysis. *Journal of the American Statistical Association*. Mar. 2012.